Math 5 – Summer 2018 Mid-Term Exam 1 30 Marks Time : 75 Minutes

[1] Write the vector $\overline{\mathrm{U}} = (xy)i + (yz)j + (x+z)k$ in the cylindrical coordinates (r, θ, z) Where $x = r\cos\theta$, $y = r\sin\theta$, $z = z$.

[2]By Gamma function, find the integral : $\int_0^\infty x\, e^{-\sqrt{x}}\, dx$

[3]Verify Green's theorem for : $\oint_C (2 - xy)\, dx + (1 + 2xy) dy$

Where C consists of : $y = x$, $x = y^2$.

[4]Verify the Gauss's theorem for the vector : $\overline{\mathrm{U}} = (2x + z)i + (x + y)j + (xy)k$ through the surface of the paraboloid $x^2 + y^2 + z = 9$, $z \geq 0$.

Dr. Mohamed Eid

Math 5 – Summer 2018 Mid-Term Exam 2 20 Marks Time : 75 Minutes

[1] Determine and sketch the image of the following region under the function $f(z) = e^z$

$0 \leq x \leq 1$, $0 \leq y \leq \frac{\pi}{2}$

[2] Show that $u(x, y) = y + e^x \sin y$ is harmonic and find its conjugate $v(x, y)$.

[3]Show that: $\underset{z=0}{\mathrm{Res}}\, f(z) = \frac{1}{2}$ where $f(z) = \frac{1}{z + \sin z}$

[4] If C is $|z - i| = 1$. Find the integrals:(a) $\oint_C \frac{\cos z}{z^2 - 9} dz$ (b) $\oint_C \frac{\sin z}{2z - i\pi} dz$

[5] If C is $|z| = 3$. Show that $\oint_C \frac{z+1}{z^2(z+2)} dz = 0$.

Dr. Mohamed Eid